Growth Diagram For Schubert RSK

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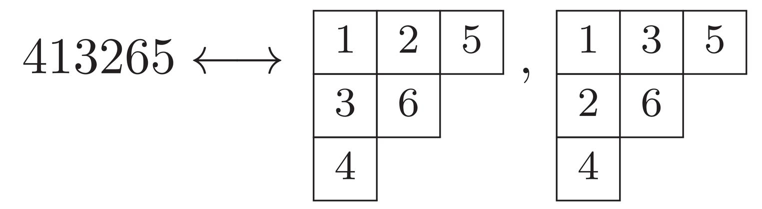
Classical RSK

Definition

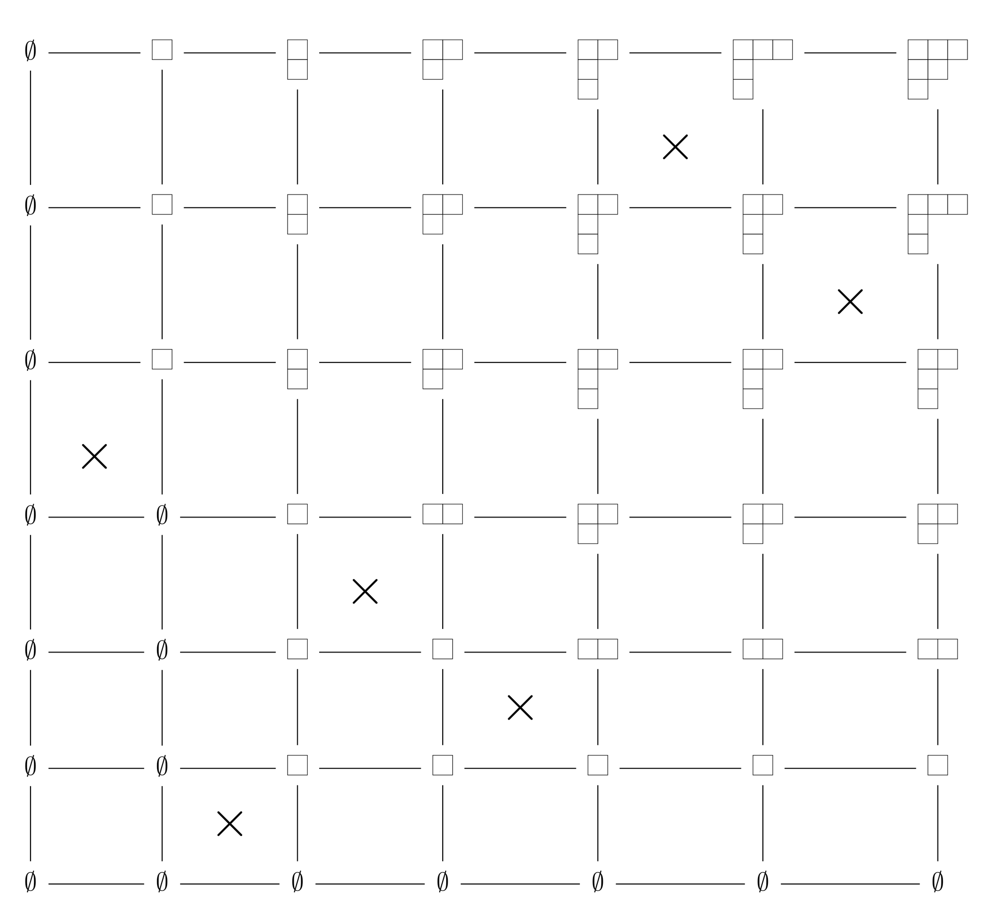
The **classical RSK correspondence** is a bijection that maps *permutations* to pairs of *standard Young tableaux* (SYT) of the same shape called *insertion tableau* and *recording tableau*.

Example



Classical RSK Growth Diagram

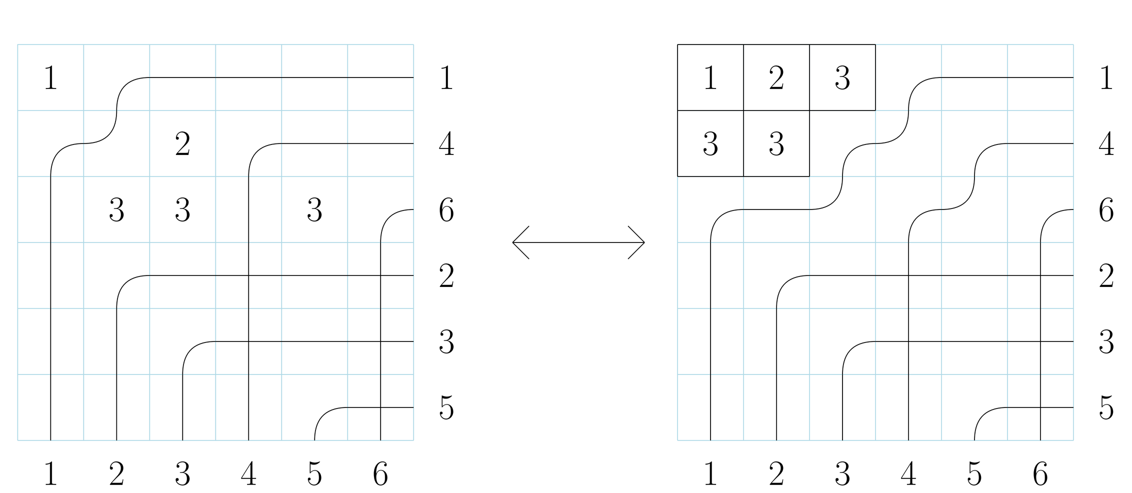
Definition [Fomin]



Properties

- Top row gives recording tableau.
- Rightmost column gives insertion tableau.

Bumpless Pipe Dreams & SSYT

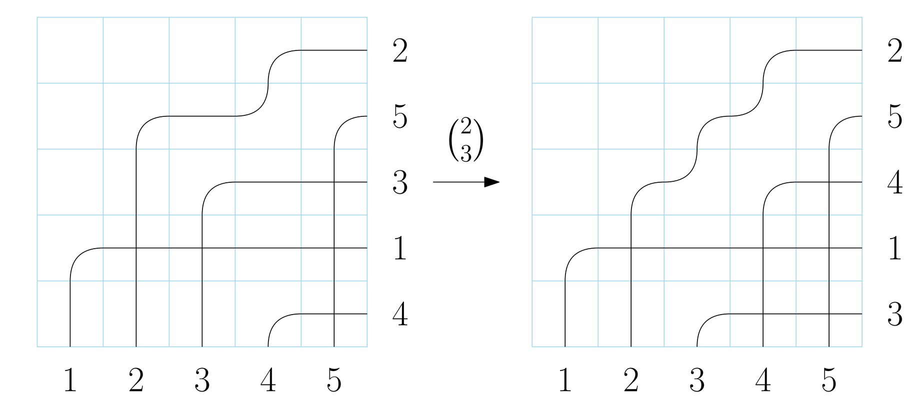


BPD of Grassmannian permutations ⟷ SSYT

Bumpless Pipe Dreams Operations

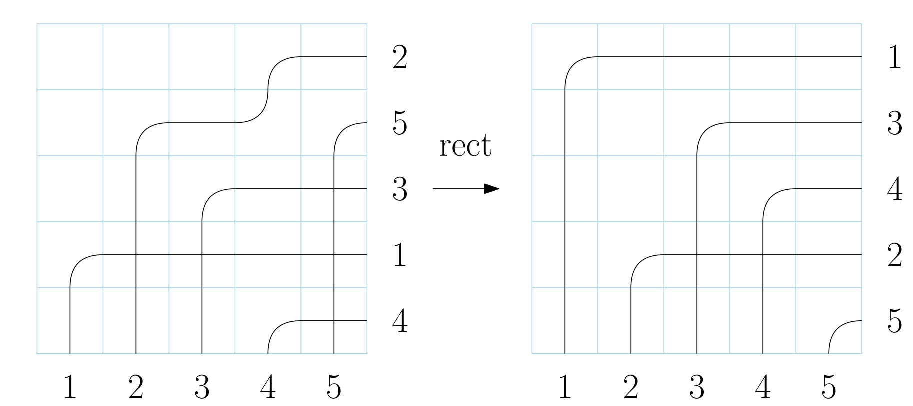
Right insertion [Huang-Pylyavskyy]

- Insert a **biletter** $\binom{b}{k}$ where $b \le k$.
- Send $D \in \mathrm{BPD}(\pi)$ to $D \leftarrow \binom{b}{k} \in \mathrm{BPD}(\pi t_{\alpha\beta})$ where $\alpha \le k < \beta$.



Rectification [Gao-Huang]

- Repeated jeu de taquin until there is no □ in the top row.
- Send $D \in \mathrm{BPD}(\pi)$ to $\mathrm{rect}(D) \in \mathrm{BPD}(s_{i_j} \dots s_{i_1}\pi)$, $i_j > \dots > i_1$.



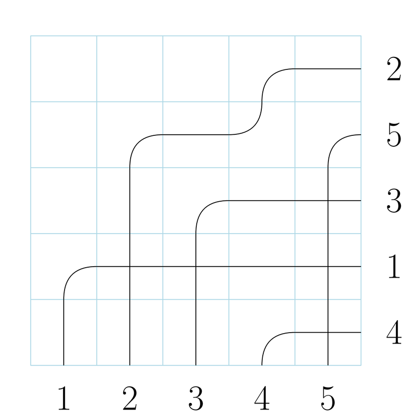
Schubert RSK

Definition [Huang-Pylyavskyy]

- A **biword** is a word of biletters $\binom{\mathbf{a}}{\mathbf{k}} = \binom{a_1, \cdots, a_\ell}{k_1, \cdots, k_\ell}$, where $k_i \ge k_{i+1}$ for each i.
- Schubert RSK maps a biword to a *insertion* BPD and a *recording chain of permutations* by right insertion.

Example

Schubert RSK sends $\binom{1\,3\,1\,2\,1}{3\,3\,2\,2\,1}$ to

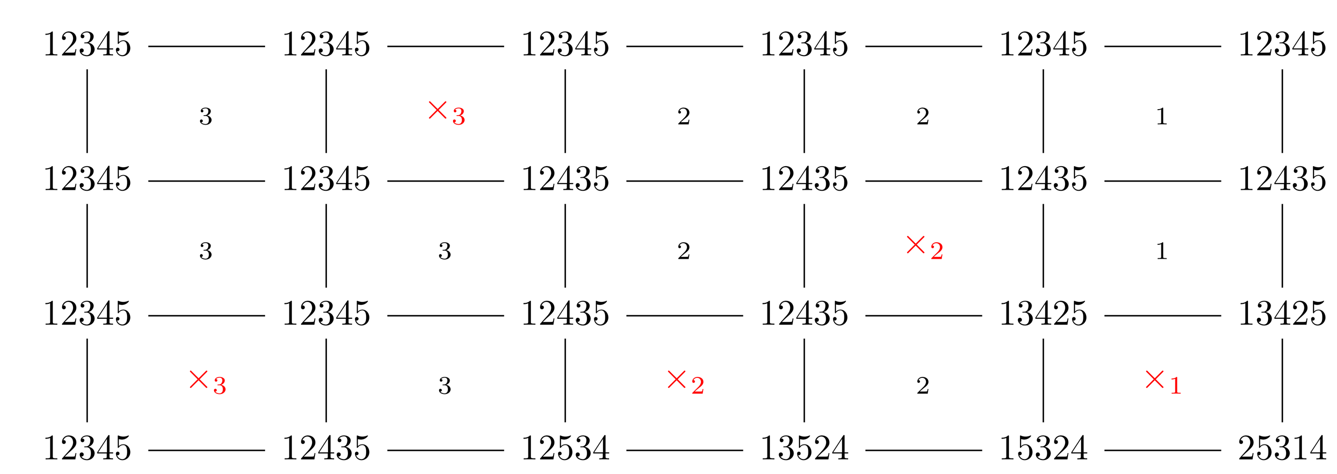


and the chain

$$12345 < 12435 < 12534 < 13524 < 15324 < 25314$$

Schubert RSK Growth Diagram

Example



Local rules

Given a square with subscript k as follows:

$$\begin{array}{ccc} \pi & - & \sigma \\ | & & | \\ \mu & - & \rho \end{array}$$

Then one can get ρ from π, μ, and σ by the following rules:

1. If there is no $\times$:
   (a) If $\pi = \sigma$ then $\rho = \mu$.
   (b) If $\pi = \mu$ then $\rho = \sigma$.
   (c) If $\pi \ne \sigma, \mu$, then $\mu = s_{i_j} \dots s_{i_1}\pi$ where $I = \{i_j > \dots > i_1\}$, and $\sigma = t_{\alpha\beta}\pi$ such that $\pi^{-1}(\alpha) \le k < \pi^{-1}(\beta)$ for some $\alpha < \beta$. Let $x := \min(I^C \cap [\alpha, \beta))$, and $A := (I^C \cap [\beta, \infty)) \cup \{x\} = \{j_1 < j_2 < \dots\}$. Then $\rho = t_{j_\ell, j_{\ell+1}}\mu$ where ℓ is the smallest index such that $\mu^{-1}(j_\ell) \le k < \mu^{-1}(j_{\ell+1})$.
2. If there is an $\times$, then $\pi = \sigma$ and $\mu = s_{i_j} \dots s_{i_1}\pi$ where $I = \{i_j > \dots > i_1\}$. Let $I^C = \{j_1 < j_2 < \dots\}$, then $\rho = t_{j_\ell, j_{\ell+1}}\mu$ where ℓ is the smallest index such that $\mu^{-1}(j_\ell) \le k < \mu^{-1}(j_{\ell+1})$.

Main Results

Theorem. *Bottom row gives recording chain.*

Theorem. *Rightmost column corresponds to insertion BPD through rectification. This recovers Gao-Huang bijection.*

Remark: Fixing k in the biword, this recovers the growth diagram for the classical RSK.

Acknowledgement

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