An extended generalization of RSK correspondence via the combinatorics of type A quiver representations

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Classical RSK correspondence

- The *classical RSK correspondence*, denoted by **RSK** [Figure 1, (Left)]:
  - We introduce **RSK** as a one-to-one correspondence from nonnegative integer matrices to pairs of semi-standard Young tableaux of the same shape.
  - Main ingredients: bi-words, *Schensted row-insertions*.
- A realization of **RSK** via *Greene–Kleitman invariants* [Figure 1, (Right)]:
  - Main ingredients: directed (sub)graph, paths, weights on collections of paths.
  - if we begin with a $n \times m$ matrix, we can display the results as a *reverse plane partition* of $\lambda = m^n$ [Figure 2]

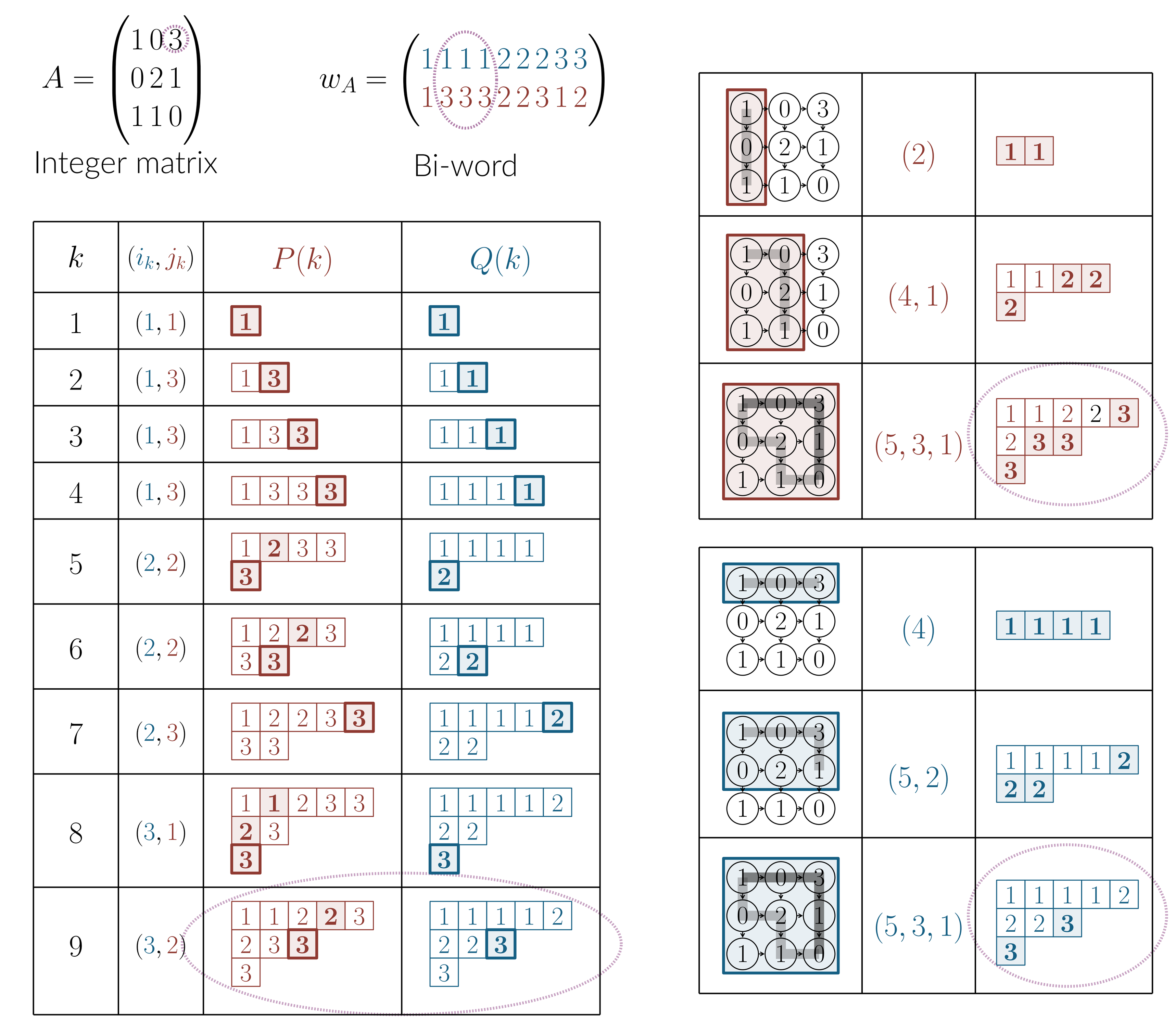

Figure 1. (Left): Illustration of the usual calculations to get $\mathbf{RSK}(A)$. (Right): Use of the Greene–Kleitman invariants to calculate $\mathbf{RSK}(A)$.

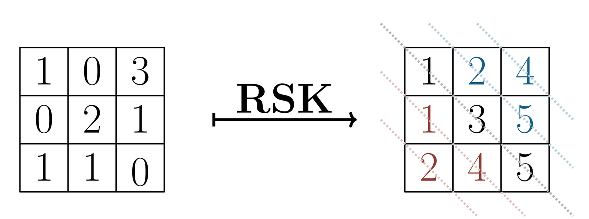

Figure 2. Results via Greene–Kleitman invariants as a reverse plane partition

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Gansner's generalized RSK

- The *Gansner's RSK correspondence*, denoted by $\mathcal{RSK}_\lambda$ [Figure 3]:
  - Fix a nonzero integer partition λ, the map $\mathcal{RSK}$ realizes a one-to-one correspondence from *filling* of λ to reverse plane partitions of shape λ.
  - Main ingredients: filling of λ, Greene–Kleitman invariant, diagonals of λ.
  - By changing the orientation from bottom to top, the analoguous map to $\mathcal{RSK}_\lambda$ coincides with the *Hillman–Grassl correspondence*.

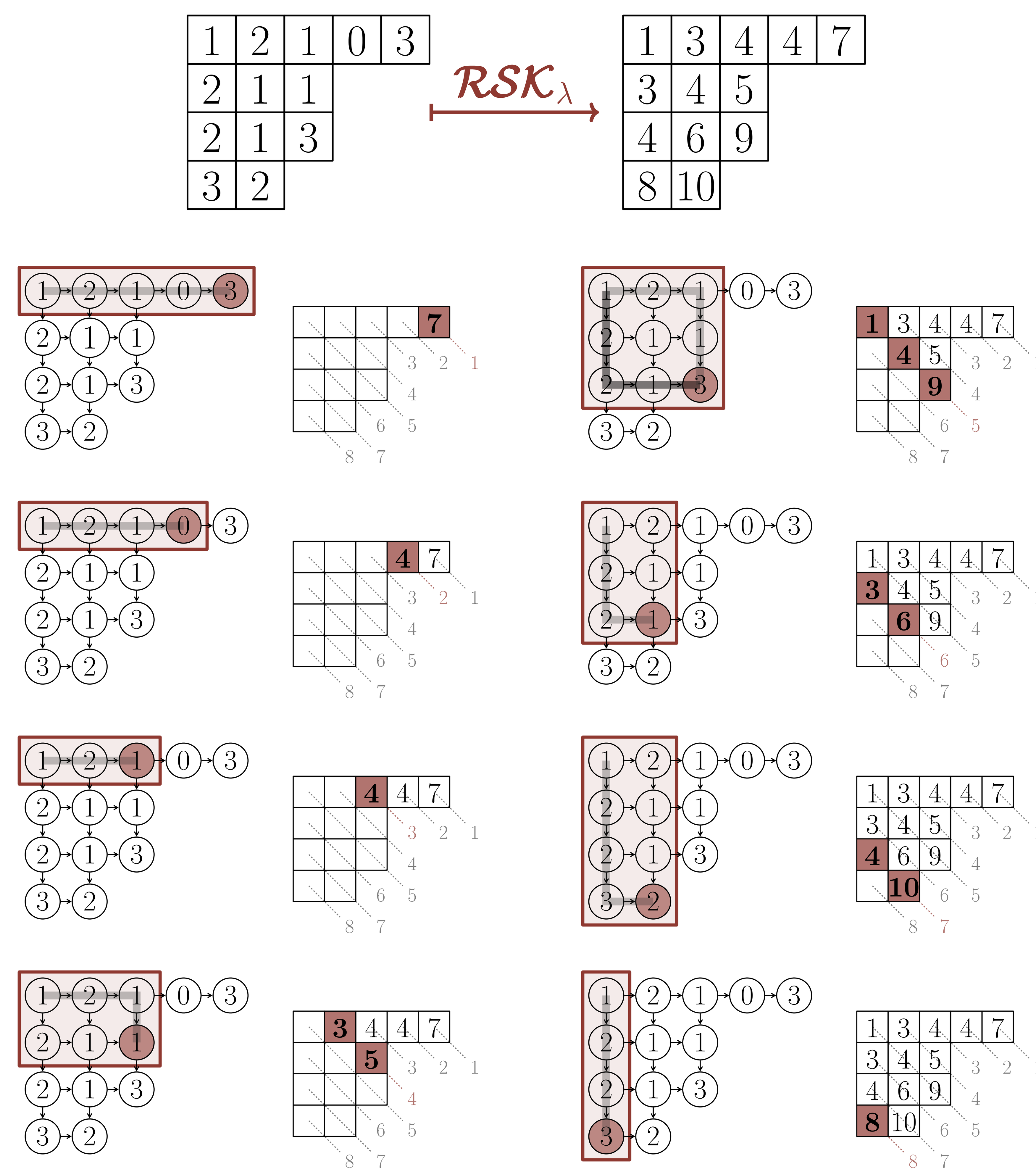

Figure 3. Explicit calculations of $\mathcal{RSK}_\lambda(f)$ for a given filling f of shape $\lambda = (5,3,3,2)$.

Extended generalization of RSK correspondence

- The *extended generalization of RSK*, denoted by $\mathcal{RSK}_{\lambda,c}$ [Figure 4]:
  - Fix an integer partition λ such that $h_\lambda(1,1) = n \geqslant 1$, we define a family of maps $(\mathcal{RSK}_{\lambda,c})_c$, parametrized by Coxeter elements $c \in \mathfrak{S}_{n+1}$, from *filling* of λ to reverse plane partitions of shape λ.
  - Main ingredients: filling of λ, *Coxeter element* $\mathbf{c}(\lambda) \in \mathfrak{S}_{n+1}$ displayed as a labelling of the boxes of λ, *Auslander–Reiten quiver of* c, Greene–Kleitman invariant, diagonals of λ.
  - Based on tools in quiver representation theory.

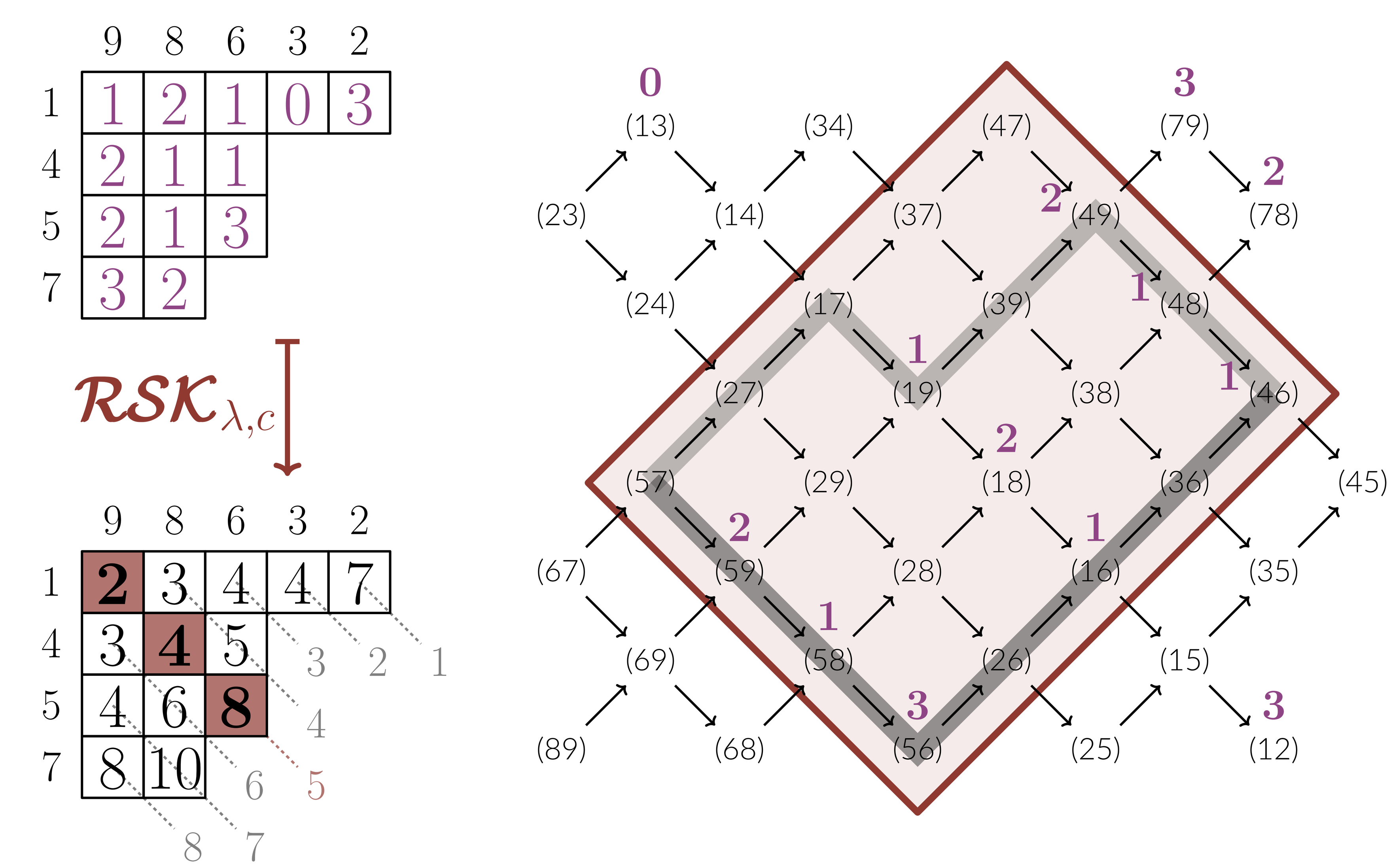

Figure 4. Explicit calculation of $\mathcal{RSK}_{\lambda,c}(f)$ for the boxes in the 5th diagonal from a filling of $\lambda = (5,3,3,2)$, with $c = (1,3,4,7,9,8,6,5,2)$

Main Results

Let λ be a nonzero integer partition, and set $n = h_\lambda(1,1)$.

- For any Coxeter element $c \in \mathfrak{S}_{n+1}$, $\mathcal{RSK}_{\lambda,c}$ establishes a one-to-one correspondance from fillings of λ to reverse plane partitions of shape λ.
- If $c = \mathbf{c}(\lambda)^{\pm 1}$, then $\mathcal{RSK}_{\lambda,c} = \mathcal{RSK}_\lambda$.
- If $c = (1,2,\ldots,n)$, then $\mathcal{RSK}_{\lambda,c}$ corresponds to the Hillman–Grassl correspondence.
- If $\lambda = m^n$, then $(\mathcal{RSK}_{\lambda,c})_c$ correspond to Dauvergne's Scramble RSKs.

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