Séminaire Lotharingien de Combinatoire **91B** (2024)
Article #76, 12 pp.

Proceedings of the 36th Conference on Formal Power Series and Algebraic Combinatorics (Bochum)

An extended generalization of RSK via the combinatorics of Type A quiver representations

Benjamin Dequêne[*12]

[1]*Laboratoire d'Algèbre de Combinatoire et d'Informatique Mathématique (UQAM, Montréal)*
[2]*Laboratoire Amiénois de Mathématiques Fondamentales et Appliquées (UPJV, Amiens)*

Abstract. The classical Robinson–Schensted–Knuth correspondence is a bijection from nonnegative integer matrices to pairs of semi-standard Young tableaux. Based on the work of, among others, Burge, Hillman, Grassl, Knuth and Gansner, it is known that a version of this correspondence gives, for any nonzero integer partition λ, a bijection from arbitrary fillings of λ to reverse plane partitions of shape λ, via Greene–Kleitman invariants. By bringing out the combinatorial aspects of our recent results on quiver representations, we construct a family of bijections from fillings of λ to reverse plane partitions of shape λ parametrized by a choice of Coxeter element in a suitable symmetric group. We recover the above version of the Robinson–Schensted–Knuth correspondence for a particular choice of Coxeter element depending on λ.

Résumé. La correspondance Robinson–Schensted–Knuth classique est une bijection partant des matrices à coefficients des entiers naturels vers les paires de tableaux de Young semi-standards. Basé sur les travaux, entre autres, de Burge, Hillman, Grassl, Knuth et Gansner, on sait qu'une version de cette correspondance donne, pour toute partage d'un entier non nulle λ, une bijection allant des remplissages arbitraires de λ vers les partitions planes renversées de forme λ, via les invariants de Greene–Kleitman. En faisant ressortir les aspects combinatoires de nos récents résultats sur les représentations de carquois, nous construisons une famille de bijections partant des remplissages de λ vers les partitions planes renversées de forme λ, paramétrées par un choix d'élément de Coxeter dans un groupe symétrique approprié. Nous récupérons la version de la correspondance Robinson–Schensted–Knuth ci-dessus pour un choix particulier d'élément de Coxeter dépendant de λ.

Keywords: Quiver representations, Robinson–Schensted–Knuth, Reverse plane partitions.

1 Introduction

The Robinson–Schensted–Knuth (RSK) correspondence is a fundamental bijection from nonnegative integer matrices to pairs of semi-standard Young tableaux of the same shape. For further details, the reader may consult the following references: [15], [5].

*dequene.benjamin@courrier.uqam.ca

Based on observations of various works of Burge [3], Hillman–Grassl [11] and Knuth [12], Gansner [6, 8] constructed a generalized version of this correspondence, via Greene–Kleitman invariants, which gives a bijection from arbitrary fillings to reverse plane partitions of the same shape.

Our paper [4] studies a representation-theoretic setting in which a version of RSK exists. In the present paper, we present an explicit, combinatorial form of the results from [4]. Given a fixed nonzero integer partition λ, we present the construction of a family of maps $(\mathrm{RSK}_{\lambda,c})_c$ from fillings of λ to reverse plane partitions of shape λ parametrized by c a Coxeter element of the symmetric group $\mathfrak{S}_n$ where $n-1$ is the hook-length of the box $(1,1)$ in λ. We can state the following result from [4].

Theorem 1. *The map* $\mathrm{RSK}_{\lambda,c}$ *gives a one-to-one correspondence from fillings of shape* λ *to reverse plane partitions of shape* λ*. Moreover, for any* λ*, there exists a unique (up to inverse) choice of* c *such that* $\mathrm{RSK}_{\lambda,c}$ *coincides with the usual* RSK*.*

No knowledge in quiver representation is required to read this abstract, except for Section 5 in which we discuss the connection with quiver representations.

2 Gansner's Ferrers Diagram RSK

In this section, we describe Gansner's correspondence explicitly.

2.1 Some vocabulary

An *integer partition* is a weakly decreasing nonnegative integer sequence $\lambda = (\lambda_n)_{n\in\mathbb{N}^*}$ with finitely many nonzero terms. The *length* of λ is the minimal $k \in \mathbb{N}$ such that $\lambda_{k+1} = 0$. We endow $(\mathbb{N}^*)^2$ with the Cartesian product order $\trianglelefteq$. The *Ferrers diagram of* λ $\mathrm{Fer}(\lambda)$ is the subset of $(\mathbb{N}^*)^2$ given by pairs (i,j) such that $i \leqslant \lambda_j$. We call any map $f : \mathrm{Fer}(\lambda) \longrightarrow \mathbb{N}$ a *filling of shape* λ . Such a filling f is a *reverse plane partition* whenever f weakly increases with respect to $\trianglelefteq$. We give an example of a reverse plane partition of shape $(5,3,3,2)$ in Figure 1.

0	3	5	5	7
1	5	5		
4	6	9		
4	10			

Figure 1: A reverse plane partitions of shape $\lambda = (5,3,3,2)$.

2.2 Greene–Kleitman invariants

Let $G = (G_0, G_1)$ be a finite directed graph, where G_0 is the set of vertices of G, and $G_1 \subset (G_0)^2$ is the set of arrows of G. Assume that G has no multi-arrows.

We see a path γ in G as a finite sequence of vertices $(v_0, \ldots, v_k)$ such that $(v_i, v_{i+1}) \in G_1$. Denote by $s(\gamma) = v_0$ its source and by $t(\gamma) = v_k$ its target. Write $\mathrm{Supp}(\gamma) = \{v_0, \ldots, v_k\}$ to denote the support of γ. For $\ell \geqslant 1$, we extend the notion of support to ℓ-tuples of paths $\underline{\gamma} = (\gamma_1, \ldots, \gamma_\ell)$ as $\mathrm{Supp}(\underline{\gamma}) = \bigcup_{i=1}^{\ell} \mathrm{Supp}(\gamma_i)$. For $\ell \geqslant 1$, write $\Pi_\ell(G)$ the set of ℓ-tuples of paths in G.

From now on, assume that G is acyclic, meaning there is no nontrivial path γ in G such that $s(\gamma) = t(\gamma)$. An *antichain* of G is any subset of vertices $\{w_1, \ldots, w_r\} \subset G_0$ such that there is no path γ in G with $s(\gamma) = w_i$ and $t(\gamma) = w_j$ for all $1 \leqslant i, j \leqslant r$ with $i \neq j$.

A *filling* of G is a map $f : G_0 \longrightarrow \mathbb{N}$. We assign to any ℓ-tuple of paths $\underline{\gamma}$ of G a *f-weight* defined by

$$\mathsf{wt}_f(\underline{\gamma}) = \sum_{v \in \mathrm{Supp}(\underline{\gamma})} f(v).$$

Set $M_0^G(f) = 0$, and for all integers $\ell \geqslant 1$, $M_\ell^G(f) = \max_{\underline{\gamma} \in \Pi_\ell(G)} \mathsf{wt}_f(\underline{\gamma})$. We define the *Greene–Kleitman invariant* of f in G as

$$\mathrm{GK}_G(f) = \left(M_\ell^G(f) - M_{\ell-1}^G(f) \right)_{\ell \geqslant 1}.$$

See Figure 2 for an explicit computation example.

Proposition 2 (Greene–Kleitman [10])**.** *Let G be a finite direct acyclic graph and f be a filling of G. The integer sequence $\mathrm{GK}_G(f)$ is an integer partition of length the maximal cardinality of an antichain in G.*

2.3 Ferrers diagram RSK

Throughout this section, we highlight Gansner's generalized version of the RSK correspondence, which gives, for any nonzero integer partition λ, a bijection from fillings of shape λ to reverse plane partitions of shape λ.

Fix a nonzero integer partition λ. Let G_λ be the oriented acyclic graph such that:

- its vertices are the elements of $\mathrm{Fer}(\lambda)$;
- its arrows are given by:
  - $(i, j) \longrightarrow (i+1, j)$ whenever $(i, j), (i+1, j) \in \mathrm{Fer}(\lambda)$;
  - $(i, j) \longrightarrow (i, j+1)$ whenever $(i, j), (i, j+1) \in \mathrm{Fer}(\lambda)$.

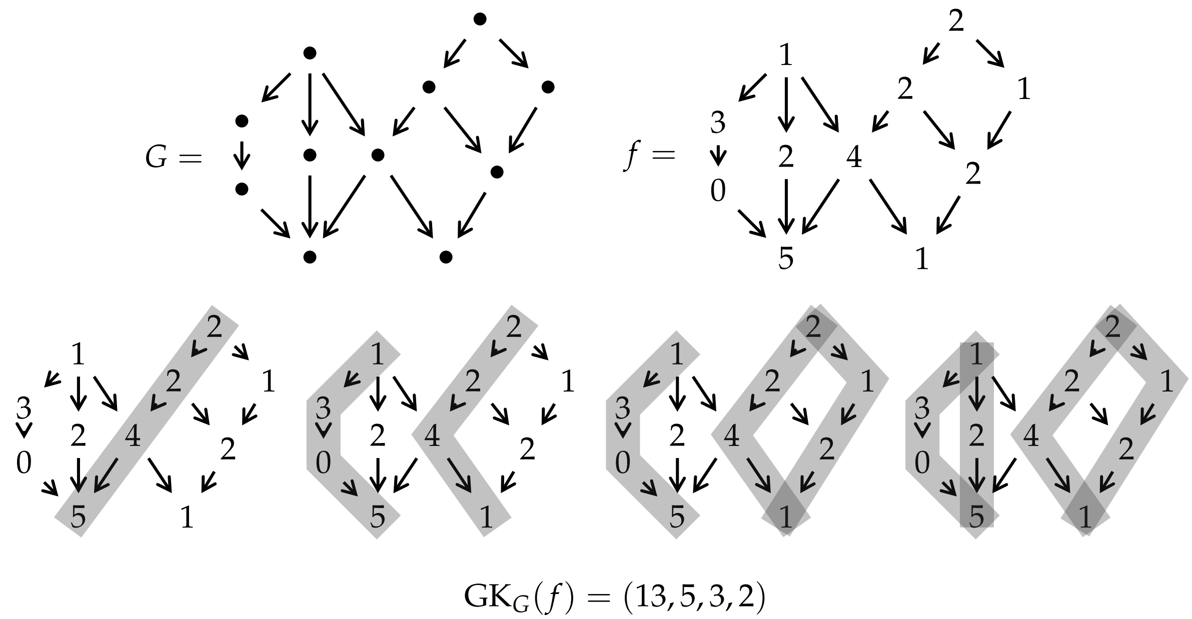

Figure 2: An example of the computation of GK_G.

For all $m \in \mathbb{Z}$, write $D_m(\lambda) = \{(i,j) \in \mathrm{Fer}(\lambda) \mid i - j + \lambda_1 = m\}$ for the mth diagonal of λ. Note that $D_m(\lambda) \neq \varnothing$ for $1 \leqslant m \leqslant h_\lambda(1,1)$, where $h_\lambda(1,1) = \#\{(i,j) \in \mathrm{Fer}(\lambda) \mid i = 1 \text{ or } j = 1\}$ denotes the *hook length of the box* $(1,1)$ *in* λ.

For each value $1 \leqslant m \leqslant h_\lambda(1,1)$, consider (u_m, v_m) the maximal element of $D_m(\lambda)$. Write $G_\lambda(m)$ for the full subgraph of G_λ given by the poset ideal generated by (u_m, v_m). Note that $G_\lambda(m)$ admits only one source $(1,1)$, and only one sink (u_m, v_m).

We define $g = \mathrm{RSK}_\lambda(f)$ to be the filling of shape λ defined by

$$\forall m \in \{1, \ldots, h_\lambda(1,1)\},\ \forall (i,j) \in D_m(\lambda), \quad g(i,j) = \mathrm{GK}_{G_\lambda(m)}(f)_{u_m - i + 1}.$$

See Figure 3 for an explicit calculation of $\mathrm{RSK}_\lambda(f)$ for a given filling of $\lambda = (5,3,3,2)$.

Theorem 3 (Gansner [8])**.** *Let λ be a nonzero integer partition. The map RSK_λ is a bijection from fillings of shape λ to reverse plane partitions of shape λ.*

Remark. If λ is a rectangle, we can recover the classical RSK. See [10] and [9, Section 6] for more details.

Moreover, a parallel can be made with Britz and Fomin's version of the RSK algorithm [2], where we compute sequences of integer partitions for an $n \times n$ nonnegative integer matrix as growth diagrams. A generalized version of RSK was also exploited by Krattenthaler [13] on polyominos. From a given filling f of shape λ, the integer partitions we can read on diagonals $D_m(\lambda)$ of $\mathrm{RSK}_\lambda(f)$ correspond precisely to the results obtained at the end of each line by using the Krattenthaler growth diagram algorithm version.

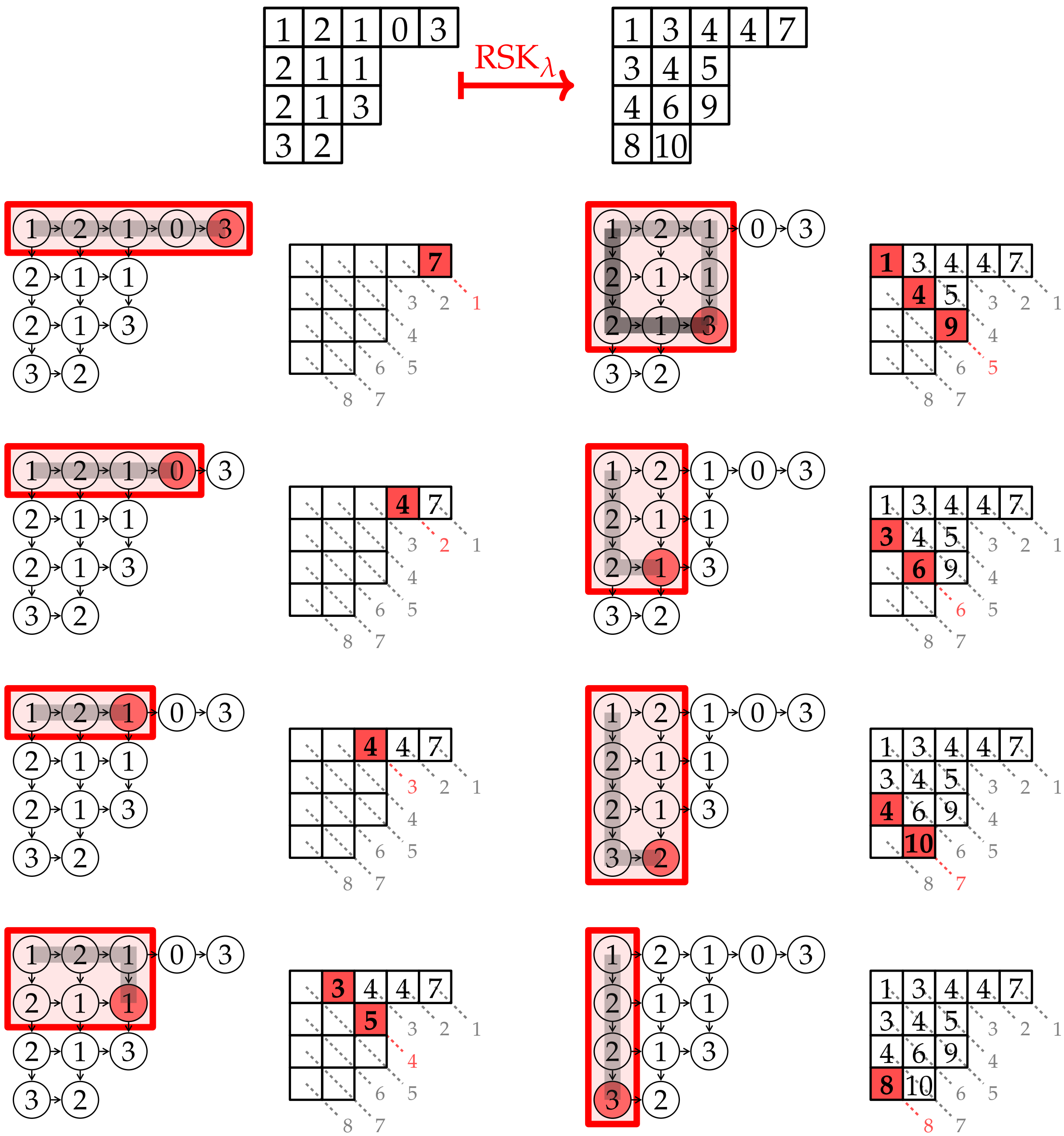

Figure 3: Explicit calculations of $\mathrm{RSK}_\lambda(f)$ for a given filling f of shape $\lambda = (5,3,3,2)$. For $1 \leqslant m \leqslant 8$, each framed subgraph corresponds to the subgraph $G_\lambda(m)$, and each filled diagonal colored in red corresponds to $\mathrm{GK}_{G_\lambda(m)}(f)$.

3 Some tools

In this section, we give the definition of some combinatorial objects that will be useful to present our generalized version of Gansner's RSK correspondence.

3.1 Interval bipartitions

An *interval bipartition* is a pair $(\mathbf{B}, \mathbf{E}) \in \mathcal{P}(\mathbb{N}^*)^2$ such that $\{\mathbf{B}, \mathbf{E}\}$ is a set partition of $\{i, \ldots, j\}$ for some $1 \leqslant i \leqslant j$. Call it *elementary* whenever $1 \in \mathbf{B}$ and $\max(\mathbf{B} \cup \mathbf{E}) \in \mathbf{E}$.

Fix $(\mathbf{B}, \mathbf{E})$ as an interval bipartition. Write $\mathbf{B} = \{b_1 < b_2 < \ldots < b_p\}$. We define the integer partition $\lambda(\mathbf{B}, \mathbf{E})$ by $\lambda(\mathbf{B}, \mathbf{E})_i = \#\{e \in \mathbf{E} \mid b_i < e\}$. If we also write $\mathbf{E} = \{e_1 < \ldots < e_q\}$, we can also describe $\lambda(\mathbf{B}, \mathbf{E})$ by its Ferrers diagram: we have $(i, j) \in \mathrm{Fer}(\lambda(\mathbf{B}, \mathbf{E}))$ whenever $b_i < e_{q-j+1}$. It allows us to label the ith row of $\mathrm{Fer}(\lambda(\mathbf{B}, \mathbf{E}))$ by b_i and the jth row by e_{q-j+1}. See Figure 4 for an example of such an object.

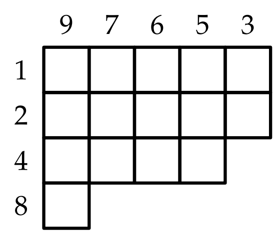

Figure 4: The (labelled) integer partition $\lambda(\mathbf{B}, \mathbf{E})$ with $\mathbf{B} = \{1, 2, 4, 8\}$ and $\mathbf{E} = \{3, 5, 6, 7, 9\}$.

Proposition 4. *For any integer partition λ, there exists an interval bipartition $(\mathbf{B}, \mathbf{E})$ such that $\lambda(\mathbf{B}, \mathbf{E}) = \lambda$. Moreover, if λ is a nonzero integer partition, there exists a unique elementary interval bipartition satisfying this property.*

3.2 (Type A) Coxeter elements

For any $n \geqslant 2$, let $\mathfrak{S}_n$ be the symmetric group on n letters. For $1 \leqslant i < j \leqslant n$, write (i, j) for the transposition exchanging i and j. For $1 \leqslant i < n$, let s_i be the adjacent transposition $(i, i+1)$. Let S be the set of the adjacent transpositions.

For any $w \in \mathfrak{S}_n$, an expression of w is a way to write w as a product of adjacent transpositions in S. The length $\ell(w)$ of w is the minimal number of adjacent transpositions in S needed to express w. Whenever, for some $1 \leqslant i < n$, $\ell(s_i w) < \ell(w)$, we say that s_i is initial in w. Similarly, we call s_i final in w whenever $\ell(w s_i) < \ell(w)$.

A *Coxeter element (of $\mathfrak{S}_n$)* is an element $c \in \mathfrak{S}_n$ which can be written as a product of all the adjacent transpositions, in some order, where each of them appears exactly once. For example, $c = s_2 s_1 s_3 s_6 s_5 s_4 s_8 s_7 = (1, 3, 4, 7, 9, 8, 6, 5, 2)$ is a Coxeter element of $\mathfrak{S}_9$.

Lemma 5. *An element $c \in \mathfrak{S}_n$ is a Coxeter element if and only if c is a long cycle which can be written as follows*

$$c = (c_1, c_2, \ldots, c_m, c_{m+1}, \ldots, c_n)$$

where $c_1 = 1 < c_2 < \ldots < c_m = n > c_{m+1} > \ldots > c_n > c_1 = 1$.

3.3 Auslander–Reiten quivers

Let $c \in \mathfrak{S}_n$ be a Coxeter element. The *Auslander–Reiten quiver of c*, denoted $\mathrm{AR}(c)$, is the oriented graph satisfying the following conditions:

- The vertices of $\mathrm{AR}(c)$ are the transpositions (i, j), with $i < j$, in $\mathfrak{S}_n$;
- The arrows of $\mathrm{AR}(c)$ are given, for all $i < j$, by
  - $(i, j) \longrightarrow (i, c(j))$ whenever $i < c(j)$;
  - $(i, j) \longrightarrow (c(i), j)$ whenever $c(i) < j$.

To construct recursively such a graph, we can first find the initial adjacent transpositions of c, which are all the sources, and step by step, using the second rule, construct the arrows and the vertices of $\mathrm{AR}(c)$ until we reach all the transpositions of $\mathfrak{S}_n$. Note that the sinks of $\mathrm{AR}(c)$ are given by the final adjacent transpositions of c. See Figure 5 for an explicit example.

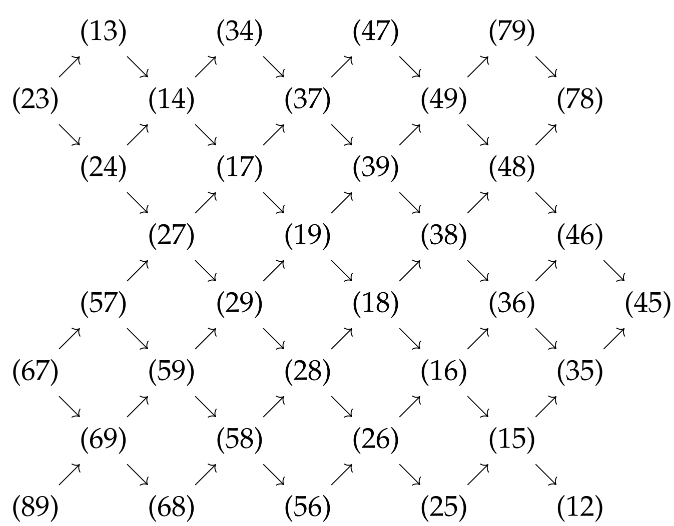

Figure 5: The Auslander–Reiten quiver of $c = (1, 3, 4, 7, 9, 8, 6, 5, 2) = s_2 s_1 s_3 s_6 s_5 s_4 s_8 s_7$.

Remark. The Auslander–Reiten quiver of a Coxeter element has a representation-theoretic meaning: briefly it corresponds to the oriented graph whose vertices are the indecomposable representations of a certain type A quiver, and whose arrows are the irreducible morphisms between them.

To see further details about Auslander-Reiten quivers of type A quivers in particular, we refer the reader to [14, Section 3.1]. To learn more about quiver representation theory, and for more in-depth knowledge on the notion of Auslander–Reiten quivers, we invite the reader to look at [1].

4 An extended generalized Ferrers diagram RSK

In the following, we describe a generalized version of RSK using (type A) Coxeter elements, and state the main result.

Let λ be a nonzero integer partition and consider $(\mathbf{B}, \mathbf{E})$ the unique elementary interval bipartition such that $\lambda(\mathbf{B}, \mathbf{E}) = \lambda$. Set $n = h_\lambda(1,1) + 1$. Let $c \in \mathfrak{S}_n$ and consider $\mathrm{AR}(c)$ its Auslander–Reiten quiver.

Recall that if $\mathbf{B} = \{b_1 < \ldots < b_p\}$ and $\mathbf{E} = \{e_1 < \ldots < e_q\}$, then $(i,j) \in \mathrm{Fer}(\lambda)$ if and only if $b_i < e_{q-j+1}$. It allows us to label each box (i,j) by a transposition $(b_i, e_{q-j+1}) \in \mathfrak{S}_n$. Thus it allows us to construct a one-to-one correspondence from fillings of shape λ to fillings of the Auslander–Reiten quiver $\mathrm{AR}(c)$ which are supported on vertices $(b,e) \in \mathbf{B} \times \mathbf{E}$ such that $b < e$. Explicitly, for any filling f of shape λ, we define $\overline{f}$ be the filling of $\mathrm{AR}(c)$ defined by $\overline{f}(b_i, e_{q-j+1}) = f(i,j)$ whenever $(i,j) \in \mathrm{Fer}(\lambda)$ and $\overline{f}(x,y) = 0$ otherwise.

As in Section 2, for $m \in \{1, \ldots, n-1\}$, let (u_m, v_m) be the maximal pair with respect of $\trianglelefteq$ in $D_m(\lambda)$. The boxes in the ideal generated by (u_m, v_m) correspond to pairs (i,j) such that $b_i \leqslant m < e_{q-j+1}$, and therefore (u_m, v_m) is the maximal pair satisfying this condition.

For each $m \in \{1, \ldots, n-1\}$, we consider the subgraph $\mathrm{AR}_m(c)$ of $\mathrm{AR}(c)$ where the vertices are the transpositions (i,j) with $i \leqslant m < j$. This subgraph has only one source and only one sink.

We define $g = \mathrm{RSK}_{\lambda,c}(f)$ to be the fillings of shape λ defined for $m \in \{1, \ldots, n-1\}$ by

$$\forall (i,j) \in D_m(\lambda), \quad g(i,j) = \mathrm{GK}_{\mathrm{AR}_m(c)}(f)_{u_m - i+1}.$$

See Figure 6 for an explicit example.

Our main result is the following.

Theorem 6. *Let λ be a nonzero integer partition. Consider $n = h_\lambda(1,1) + 1$. Let $c \in \mathfrak{S}_n$ be a Coxeter element. The map $\mathrm{RSK}_{\lambda,c}$ gives a one-to-one correspondence from fillings of shape λ to reverse plane partitions of shape λ.*

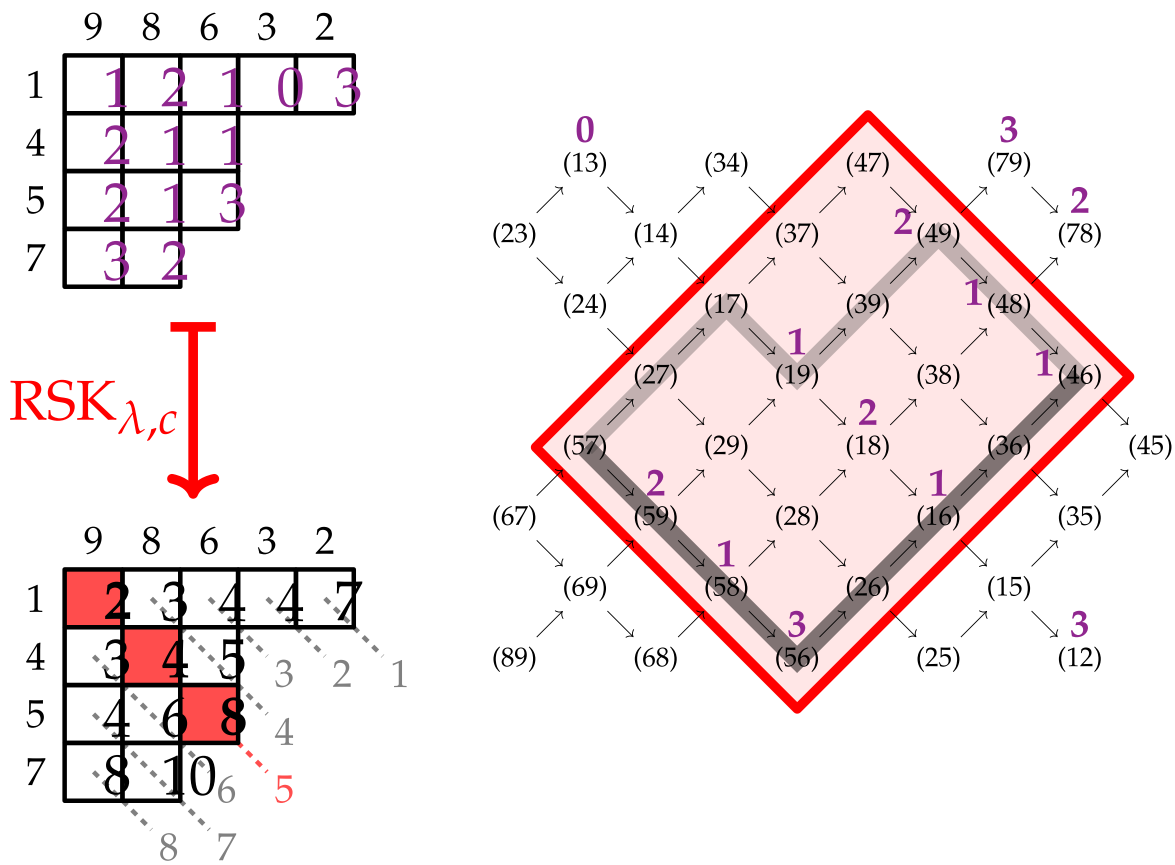

Figure 6: Explicit calculation of $\text{RSK}_{\lambda,c}(f)$ for the boxes in $D_5(\lambda)$ from a filling of $\lambda = (5,3,3,2)$, with $c = (1,3,4,7,9,8,6,5,2)$

The following result shows that we extended the RSK correspondence.

Proposition 7. *Let λ be a nonzero integer partition. Consider $n = h_\lambda(1,1) + 1$ and $(\mathbf{B},\mathbf{E})$ be the only elementary interval bipartition such that $\lambda(\mathbf{B},\mathbf{E}) = \lambda$. Let $c \in \mathfrak{S}_n$ be the Coxeter element such that*

- *for $i \in \{1,\dots,n-1\}$, $(i,i+1)$ is final in c if and only if $i \in \mathbf{B}$ and $i+1 \in \mathbf{E}$;*
- *for $i \in \{2,\dots,n-2\}$, $(i,i+1)$ is initial in c if and only if $i \in \mathbf{E}$ and $i+1 \in \mathbf{B}$.*

Then $\text{RSK}_{\lambda,c} = \text{RSK}_\lambda$. Moreover, c and c^{-1} are the unique Coxeter element of $\mathfrak{S}_n$ satisfying this property.

Remark. Gansner's RSK for a fixed integer partition λ admits a local description in terms of toggles on G_λ. Based on the proof given in [4], for $c = (1,2,\dots,n)$, we can give a local description in terms of toggles on $\mathrm{AR}(c)$. However, more works need to be done for a general choice of c, as this local description does not extend naturally.

5 Some words about quiver representation theory

This section aims to give a dictionary to link the result from [4] with Theorem 6.

Fix $Q = (Q_0, Q_1)$ a type A quiver. A *finite dimensional representation E of Q over $\mathbb{C}$* is an assignment of a finite dimensional $\mathbb{C}$-vector space E_q to each vertex q of Q, and an assignment of a $\mathbb{C}$-linear transformation $E_\alpha : E_i \longrightarrow E_j$ to each arrow $\alpha : i \to j$ of Q. For two representations E and F, a morphism $\phi : E \longrightarrow F$ is the data of a $\mathbb{C}$-linear map ϕ_q for each vertex q of Q such that for any arrow $\alpha : i \to j$, $\phi_j E_\alpha = F_\alpha \phi_i$. Denote by $\mathrm{rep}_{\mathbb{K}}(Q)$ the category consisting of the representations of Q.

Any representation E of Q can be uniquely decomposed into a direct sum of copies of indecomposable representations up to isomorphism. Thus, we can consider the invariant which counts the number of indecomposable summands of each isomorphism class in E. Write it $\mathrm{Mult}(E)$.

In [9], A. Garver, R. Patrias and H. Thomas introduce a new invariant of quiver representations, called the generic Jordan form data. For any representation E of Q, write $\mathrm{GenJF}(E)$ for the generic Jordan form data of E. This data encodes the generic behavior of a nilpotent endomorphism $N = (N_q)_{q \in Q_0}$ of the representation via the size of the Jordan blocks of each N_q. In some subcategories, the representation can be recovered from this invariant up to isomorphism.

They also show that the map from Mult to GenJF generalizes the RSK correspondence for type A quivers, using Gansner's previous work [7].

As this map is bijective, if we restrict it to the representation in some subcategories $\mathscr{C}$, one can be interested to get an explicit way to invert it. An algebraic method developed in [9] asks the subcategory $\mathscr{C}$ to satisfy the following property. For any $E \in \mathscr{C}$, there exists a dense open set Ω (in the Zariski topology) in the set of representations admitting a nilpotent endomorphism with Jordan forms encoded by $\mathrm{GenJF}(E)$ such that any $F \in \Omega$ is isomorphic to E. Such a subcategory is said to be *canonically Jordan recoverable (CJR)*.

More recently, in [4], we gave a combinatorial characterization of all the CJR subcategories of representations of Q, substancially enlarging the family of subcategories for which GenJF is a complete invariant given by [9]. The maximal such subcategories can be described thanks to the elementary interval partitions $(\mathbf{B}, \mathbf{E})$ of $\{1, \ldots, n+1\}$.

The following table compares the representation-theoretic tools used in [4] and the combinatorial tools used to describe our generalized RSK.

Combinatorial tools	**Representation-theoretic tools**
Coxeter element of $\mathfrak{S}_n$	Orientation of an A_{n-1} type quiver Q
Transposition in $\mathfrak{S}_n$	Indecomposable representation in $\operatorname{rep}_{\mathbb{C}}(Q)$
AR quiver of c	AR quiver of $\operatorname{rep}_{\mathbb{C}}(Q)$
Integer partition λ with $h_\lambda(1,1) = n-1$	CJR subcategory $\mathscr{C}$ of $\operatorname{rep}_{\mathbb{C}}(Q)$
Filling of λ	$\operatorname{Mult}(E)$ for some $E \in \mathscr{C}$
Reverse plane partition of λ	$\operatorname{GenJF}(E)$ for E in $\mathscr{C}$.

Acknowledgements

I acknowledge the Institut des Sciences Mathématiques of Canada for its partial funding support. I thank my Ph.D. supervisor, Hugh Thomas, for all our discussions on this subject, his helpful advice, and his support throughout my thesis work, and even more.

References

[1] I. Assem, D. Simson, and A. Skowroński. *Elements of the Representation Theory of Associative Algebras: Techniques of Representation Theory*. Vol. 1. London Mathematical Society Student Texts. Cambridge University Press, 2006. DOI.

[2] T. Britz and S. Fomin. "Finite Posets and Ferrers Shapes". *Advances in Mathematics* **158**.1 (2001), pp. 86–127. DOI.

[3] W. H. Burge. "Four Correspondences Between Graphs and Generalized Young Tableaux". *Journal of Combinatorial Theory, Series A* **17** (1972), pp. 12–30.

[4] B. Dequêne. "Canonically Jordan recoverable categories for modules over the path algebra of A_n type quivers". 2023. arXiv:2308.16626.

[5] W. Fulton. *Young Tableaux: With Applications to Representation Theory and Geometry*. London Mathematical Society Student Texts. Cambridge University Press, 1996. DOI.

[6] E. Gansner. "Matrix correspondences of plane partitions". *Pacific Journal of Mathematics* **92** (Feb. 1981), pp. 295–315. DOI.

[7] E. R. Gansner. "Acyclic Digraphs, Young Tableaux and Nilpotent Matrices". *SIAM Journal on Algebraic Discrete Methods* **2**.4 (1981), pp. 429–440. DOI.

[8] E. R. Gansner. "The Hillman-Grassl correspondence and the enumeration of reverse plane partitions". *Journal of Combinatorial Theory, Series A* **30**.1 (1981), pp. 71–89. DOI.

[9] A. Garver, R. Patrias, and H. Thomas. "Minuscule reverse plane partitions via quiver representations". *Selecta Mathematica* **29**.3 (2023), p. 37. DOI.

[10] C. Greene and D. J. Kleitman. "The structure of Sperner k-families". *Journal of Combinatorial Theory, Series A* **20**.1 (1976), pp. 41–68. DOI.

[11] A. P. Hillman and R. M. Grassl. "Reverse plane partitions and tableau hook numbers". *Journal of Combinatorial Theory, Series A* **21** (1976), pp. 216–221.

[12] D. E. Knuth. "Permutations, matrices, and generalized Young tableaux." *Pacific J. Math* **34**.3 (1970), pp. 709–727.

[13] C. Krattenthaler. "Growth diagrams, and increasing and decreasing chains in fillings of Ferrers shapes". *Advances in Applied Mathematics* **37**.3 (2006). Special Issue in honor of Amitai Regev on his 65th Birthday, pp. 404–431. DOI.

[14] R. Schiffler. *Quiver Representations by Ralf Schiffler.* 1st ed. 2014. CMS Books in Mathematics, Ouvrages de mathématiques de la SMC. Cham: Springer International Publishing, 2014.

[15] R. P. Stanley. *Enumerative Combinatorics*. Vol. 2. Cambridge University Press, 1999. Link.